

Theoretical Concepts in Physics

An Alternative View of
Theoretical Reasoning in Physics

MALCOLM S. LONGAIR

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1 Introduction

1.1 An explanation for the reader

This book is for students who love physics and theoretical physics. It arises from the dichotomy which, in my view, pervades most attempts to teach the ideal course in physics. On the one hand, there is the way in which university teachers present the subject in lecture courses and examples classes. On the other hand, there is the way in which we actually practise the discipline as professional physicists. In my experience, there is often little relation between these activities. This is a great misfortune because students are then rarely exposed to their lecturers when they are practising their profession as physicists.

There are good reasons, of course, why the standard lecture course has evolved into its present form. First of all, physics and theoretical physics are not particularly easy subjects and it is important to set out the fundamentals in as clear and systematic a manner as possible. It is absolutely essential that students acquire a firm grounding in the basic techniques and concepts of physics. But we should not confuse this process with that of doing real physics. Standard lecture courses in physics and its associated mathematics are basically ‘five-finger’ exercises, designed to develop technique and understanding. But such exercises are very different from a performance of the *Hammerklavier* sonata at the Royal Festival Hall. You are only doing physics or theoretical physics when the answers *really* matter – when your reputation as a scientist hangs upon being able to reason correctly in a research context or, in more practical terms, when your ability in undertaking original research determines whether you are employable, or whether your research grant is renewed. This is a quite different process from working through drill exercises, for which answers are available at the back of the book.

Second, there is so much material which lecturers feel they have to include in their courses that all physics syllabuses are seriously overloaded. There is generally little time left for sitting back and asking ‘What is this all about?’ Indeed, the technical aspects of the subject, which are themselves fascinating, can become so totally absorbing that it is generally left to the students to find out for themselves many essential truths about physics.

Let me list some aspects of the practice of physics which can be missed in our teaching but which, I believe, are essential aspects of the way in which we carry it out as professionals.

(i) A series of lecture courses is by its nature a modular exercise. It is only too easy to lose a *global view* of the whole subject. Professionals use the whole of physics in tackling problems and there is no artificial distinction between thermal physics, optics, mechanics, electromagnetism, quantum mechanics and so on.

(ii) A corollary of this is that in physics any problem can normally be tackled and solved in a variety of different ways. Often *there is no single 'best way' of solving a problem*; much deeper insights into how the physics works can be obtained if the problem is approached from very different standpoints, for example, from thermodynamics, electromagnetism, quantum theory and so on.

(iii) How problems are tackled and how one thinks about physics are rather personal matters. No two professional physicists think in exactly the same way because we all have different experiences of using the tools of physics in a research context. When we come to write down the relevant equations and solve them, however, we should come to the same answers. The *individual physicist's response to the subject* is an integral part of the way in which physics is taught and practised, to a much greater extent than students or the lecturers themselves would like to believe. But it is the diversity of different lecturers' approaches to physics which provides insight into the nature of the mental processes by which they understand their subject. I remember vividly a splendid lecture by my colleague Douglas Gough summarising a colloquium in Vienna entitled *Inside the Stars*, in which he concluded with the following wonderful paragraph:

'I believe that one should never approach a new scientific problem with an unbiased mind. Without prior knowledge of the answer, how is one to know whether one has obtained the right result? But with prior knowledge, on the other hand, one can usually correct one's observations or one's theory until the outcome is correct... However, there are rare occasions on which, no matter how hard one tries, one cannot arrive at the correct result. Once one has exhausted all possibilities for error, one is finally forced to abandon a prejudice, and redefine what one means by 'correct'. So painful is the experience that one does not forget it. That subsequent replacing of the old prejudice by a new one is what constitutes a gain in real knowledge. And that is what we, as scientists, continually pursue.'¹

In fact, Douglas's dictum is the foundation of the process of discovery in research. All of us have different prejudices and personal opinions about what the solutions to problems might be and it is this diversity of approach which leads to new understandings.

(iv) Another potential victim of the standard lecture course is an appreciation of what it feels like to be involved in *research at the frontiers of knowledge*. Lecturers are always at their best when they reach the part of the course where they can slip in the things which excite them in their research work. For a few moments, the lecturer is transformed from a teacher into a research scientist and then the students see the real physicist at work.

(v) It is often difficult to convey the *sheer excitement of the processes of research and discovery in physics* and yet these are the very reasons that most of us get so enthusiastic about our research; once you are into a challenging research problem, it will not go away. The caricature of the 'mad' scientist is not wholly a myth in that, in carrying out frontier research, it is almost essential to become at times totally absorbed in the problems to the virtual exclusion of the cares of normal life. The biographies of many of the greatest scientists illustrate the extraordinary powers of concentration which they possessed – the examples of Newton and Faraday spring immediately to mind as physicists who, once embarked upon a fertile seam of research, would work unrelentingly until the inspiration was exhausted. All professional physicists have experience of this total intellectual commitment at much

more modest levels of achievement and it is only later that, on reflection, we regard these as among our best research experiences. Yet some students complete a physics course without really being aware of what it is that drives us on.

(vi) Much of this excitement can be conveyed through examples selected from the *history* of some of the great discoveries in physics and yet these seldom appear in our courses. The reasons are not difficult to fathom. First of all, there is just not time to do justice to the material. Second, it is not a trivial matter to establish the relevant historical material – physics has created its own mythologies as much as any other subject. Third, nowadays the history and philosophy of science are generally taught as wholly separate disciplines from physics and theoretical physics. My view is that an appreciation of some historical case studies can provide invaluable insight into the processes of research and discovery in physics and of the intellectual framework within which they took place. In these historical case studies, we recognise parallels with our own research experience.

(vii) In these historical examples, key factors familiar to all professional physicists are the central roles of *hard work*, *experience* and, perhaps most important of all, *intuition*. Many of the most successful physicists depend very heavily upon intuition gained through their wide experience and a great deal of hard work in physics and theoretical physics. It would be marvellous if experience could be taught, but I am convinced that it is something which can only be achieved by dedicated hard work. We all remember our mistakes and the blind alleys we have entered and these teach us as much about physics as our successes. Intuition is potentially a dangerous tool because one can make some very bad blunders by relying on it too heavily in frontier areas of physics. Yet it is certainly the source of many of the greatest discoveries in physics. These were not achieved using five-finger exercise techniques, but involved leaps of the imagination which transcended known physics.

(viii) These considerations bring us close to what I regard as the central core of our experience as physicists and theoretical physicists. There is an essential element of *creativity* which is not so different from creativity in the arts. The leaps of imagination involved in discovering, say, Newton's laws of motion, Maxwell's equations, relativity and quantum theory are not so different in essence from the creations of the greatest artists, musicians, writers and so on. The basic differences are that physicists must be creative within a very strict set of rules and that their theories should be testable by confrontation with experiment and observation. Very few of us indeed attain the almost superhuman level of intuition involved in discovering a wholly new physical theory, but we are driven by the same creative urge. Each small step we make contributes to the sum of our understanding of the nature of our physical universe. All of us in our own way tread in regions where no one has passed before.

(ix) The imagination and creativity involved in the very best experimental and theoretical physics result unquestionably in a real sense of *beauty*. The great achievements of physics evoke in me, at least, the same type of response that one finds with great works of art. I suspect that many of us feel the same way about physics but are generally too embarrassed to admit it. This is a pity because the achievements of experimental and theoretical physics rank among the very peaks of human endeavour. I think it is important to tell students when I find a piece of physics particularly beautiful – and there are many examples of this.

When I teach such topics, I experience the same process of rediscovery as on listening to a familiar piece of classical music – one’s umpteenth hearing of the *Eroica* symphony or of *Le Sacre du printemps*. I am sure students should know about this.

(x) Finally, physics is *great fun*. The standard lecture course with its concentration on technique can miss so much of the enjoyment and stimulation of the subject. It is essential to convey our enthusiasm for physics. Although physics finds practical application in a myriad of different areas, I am quite unashamed about promoting it for its own sake – if any apologia for this position is necessary, it is that in coming to a real understanding of our physical world our intellectual and imaginative powers are stretched to their very limits.

In this book, I adopt a very different approach to theoretical reasoning in physics from that of the standard textbook. The emphasis is upon the genius and excitement of the discovery of new insights into the laws of physics, much of it through a careful analysis of historical case studies. But my aims are more than simply attempting to redress the balance in the way in which physics is presented. Some of these further aims can be appreciated from the history of how this book came about.

1.2 How this book came about

The origin of this book can be traced to discussions in the Cambridge Physics Department in the mid-1970s among those who were involved in teaching theoretically biased undergraduate courses. There was a feeling that the syllabuses lacked coherence from the theoretical perspective and that the students were not quite clear about the scope of *physics* as opposed to *theoretical physics*. Are they really such different topics?

As our ideas evolved, it became apparent that a discussion of these ideas would be of value to all final-year students. A course entitled ‘Theoretical concepts in physics’ was therefore designed, to be given in the summer term in July and August to undergraduates entering their final year. It was to be strictly non-examinable and entirely optional. Students obtained no credit from having attended the course beyond an increased appreciation of physics and theoretical physics. I was invited to give the first presentation of this course of lectures, with the considerable challenge of attracting students to 9.00 a.m. lectures on Mondays, Wednesdays and Fridays during the most glorious summer months in Cambridge.

We agreed that the course should contain discussion of the following elements:

- (a) *the interaction between experiment and theory*. Particular stress would be laid upon the importance of experiment and, in particular, novel technology in leading to theoretical advances;
- (b) the importance of having available the *appropriate mathematical tools for tackling theoretical problems*;
- (c) *the theoretical background to the basic concepts of modern physics*, emphasising underlying themes such as *symmetry, conservation, invariance and so on*;
- (d) *the role of approximations and models in physics*;

- (e) *the analysis of real scientific papers in theoretical physics*, providing insight into how professional physicists tackle real problems.

I decided to approach these topics through a series of case studies designed to illuminate these different aspects of physics and theoretical physics. We also had the following aim:

- (f) *to consolidate and revise* many of the basic physical concepts which all final-year undergraduates can reasonably be expected to have at their fingertips.

Finally, I wanted the course

- (g) *to convey my own personal enthusiasm for physics and theoretical physics*. My own research is in high-energy astrophysics and astrophysical cosmology, but I remain a physicist at heart: my own view is that astronomy, astrophysics and cosmology are no more than subsets of physics, but applied to the Universe on the large scale. My own enthusiasm results from being involved in astrophysical and cosmological research at the very limits of our understanding of the Universe. I am one of the very lucky generation who began research in astrophysics in the early 1960s and who have witnessed the amazing revolutions which have taken place in our understanding of all aspects of the physics of the Universe. But similar sentiments could be expressed about all areas of physics. The subject is not a dead, pedagogic discipline, the only object of which is to provide examination questions for students. It is an active, extensive subject in a robust state of good health.

After giving the course for four summers, I moved to Edinburgh where the first edition of this book was written. I returned to Cambridge in 1991 and, from 1998, have presented the course, now called ‘*Concepts in physics*,’ to the third-year undergraduates. In this second edition, I have introduced new case studies and elaborated many of the ideas which stimulated the original course. To make the coverage more complete and enhance its usefulness to students, I have included material from examples classes in mathematical physics as well as material arising from my experience of lecturing on essentially the whole of physics. Further explanations of areas in which it is my experience that students find help valuable are included in chapter appendices.

1.3 A warning to the reader

The reader should be warned of two things. First, this is necessarily a *personal view of the subject*. It is intentionally designed to emphasise items (i) to (x) and (a) to (g) – in other words, to emphasise all those aspects which tend to be squeezed out of physics courses because of lack of time.

Second, and even more important, this set of case studies is not a textbook. It is certainly *not* a substitute for the systematic development of these topics through standard physics and mathematics courses. You should regard this book as a supplement to the standard courses, but one which I hope may enhance your understanding, appreciation and enjoyment of physics.

1.4 The nature of physics and theoretical physics

Let us begin by making a formal statement about the basis of our scientific endeavour. The natural sciences aim to give a logical and systematic account of natural phenomena and to enable us to predict from our past experience to new circumstances. *Theory* is the formal basis for such arguments; it need not necessarily be expressed in mathematical language, but the latter gives us the most powerful and general method of reasoning we possess. Therefore, wherever possible we attempt to secure *data* in a form that can be handled *mathematically*. There are two immediate consequences for theory in physics.

The first consequence is that the basis of all physics is *experimental data* and the necessity that these data be in *quantified form*. Some would like to believe that the whole of theoretical physics could be produced by pure reason, but they are doomed to failure from the outset. The great achievements of theoretical physics have been solidly based upon the achievements of experimental physics, which provides powerful constraints upon physical theory. Every theoretical physicist should therefore have a good and sympathetic understanding of the methods of experimental physics, not only so that theory can be confronted with experiment in a meaningful way but also so that new experiments can be proposed which are realisable and which can discriminate between rival theories.

The second consequence, as stated earlier, is that we must have adequate *mathematical tools* with which to tackle the problems we need to solve. Historically, the mathematics and the experiments have not always been in step. Sometimes the mathematics has been available but the experimental methods needed to test the theory have been unavailable. In other cases, the opposite has been true – new mathematical tools have had to be developed to describe the results of experiment.

Mathematics is central to reasoning in physics but we should beware of treating it as the whole physical content of theory. Let me reproduce some words from the reminiscences of Paul Dirac about his attitude to mathematics and theoretical physics. Dirac sought mathematical beauty in all his work. For example, on the one hand he writes:

Of all the physicists I met, I think Schrödinger was the one that I felt to be most closely similar to myself . . . I believe the reason for this is that Schrödinger and I both had a very strong appreciation of mathematical beauty and this dominated all our work. It was a sort of act of faith with us that any equations which describe fundamental laws of Nature must have great mathematical beauty in them. It was a very profitable religion to hold and can be considered as the basis of much of our success.²

On the other hand, earlier he writes:

I completed my [undergraduate] course in engineering and I would like to try to explain the effect of this engineering training on me. Previously, I was interested only in exact equations. It seemed to me that if one worked with approximations there was an intolerable ugliness in one's work and I very much wanted to preserve mathematical beauty. Well, the engineering training which I received did teach me to tolerate approximations and I was able to see that even theories based upon approximations could have a considerable amount of beauty in them.

There was this whole change of outlook and also another, which was perhaps brought on by the theory of relativity. I had started off believing that there were some exact laws of Nature and that all we had to do was to work out the consequences of these exact laws. Typical of these were Newton's

laws of motion. Now, we learned that Newton's laws of motion were not exact, only approximations, and I began to infer that maybe all the laws of nature were only approximations . . .

I think that if I had not had this engineering training, I should not have had any success with the kind of work I did later on because it was really necessary to get away from the point of view that one should only deal with exact equations and that one should deal only with results which could be deduced logically from known exact laws which one accepted, in which one had implicit faith. Engineers were concerned only in getting equations which were useful for describing nature. They did not very much mind how the equations were obtained. . . .

And that led me of course to the view that this outlook was really the best outlook to have. We wanted a description of nature. We wanted the equations which would describe nature and the best we could hope for was, usually,* approximate equations and we would have to reconcile ourselves to an absence of strict logic.³

These are very important and profound sentiments which should be familiar to the reader. There is really no strictly logical way in which we can formulate theory – we are continually approximating and using experiment to keep us on the right track. Note that Dirac was describing theoretical physics at its very highest level – concepts like Newton's laws of motion, special and general relativity, Schrödinger's equation and the Dirac equation are the *very summits of achievement of theoretical physics* and very few can work creatively at that level. The same sentiments apply, however, in their various ways to all aspects of research as soon as we attempt to model quantitatively the natural world.

Most of us are concerned with applying and testing known laws to physical situations in which their application has not previously been possible, or foreseen, and we often have to make numerous approximations to make the problem tractable. The essence of our training as physicists is to develop confidence in our physical understanding of physics so that, when we are faced with a completely new problem, we can use our experience and intuition to recognise the most fruitful ways forward.

1.5 The influence of our environment

1.5.1 *The international scene*

It is important to realise not only that all physicists are individuals with their own prejudices but also that these prejudices are strongly influenced by the tradition within which they have studied physics. I have had experience of working in a number of different countries, particularly in the USA and the former Soviet Union, and the different scientific traditions can be appreciated vividly in the marked difference in approach of physicists to research problems. This has added greatly to my understanding and appreciation of physics.

An example of a distinctively British feature of physics is the tradition of *model building*, to which we will return on several occasions. Model building seems to have been an especially British trait during the nineteenth and early twentieth centuries. The works of Faraday and Maxwell are full of models, as we will see, and at the beginning of the twentieth century, the variety of models for atoms was quite bewildering. The J.J. Thomson

* Editorial commas.

‘plum-pudding’ model of the atom is perhaps one of the more famous examples, but it is just the tip of the iceberg. Thomson was quite straightforward about the importance of model building:

The question as to which particular method of illustration the student should adopt is for many purposes of secondary importance provided that he does adopt one.⁴

Thomson’s assertion is splendidly illustrated by Heilbron’s *Lectures on the History of Atomic Physics 1900–1920*.⁵ The modelling approach is very different from the continental European tradition of theoretical physics – we find Poincaré remarking that ‘The first time a French reader opens Maxwell’s book, a feeling of discomfort, and often even of mistrust, is at first mingled with his admiration . . .’.⁶ According to Hertz, Kirchhoff was heard to remark that he found it painful to see atoms and their vibrations wilfully stuck in the middle of a theoretical discussion.⁷ It was reported to me after a lecture in Paris that one of the senior professors had commented that my presentation had not been ‘sufficiently Cartesian’. I believe the British tradition of model-building is alive and well. I can certainly vouch for the fact that, when I think about some topic in physics or astrophysics, I generally have some picture, or model, in my mind rather than an abstract or mathematical idea.

I believe the development of *physical insight* is an integral part of the model-building tradition. The ability to guess correctly what will happen in a new physical situation without having to write down all the mathematics is a very useful talent and most of us develop it with time. It must be emphasised, however, that having physical insight is no substitute for producing exact mathematical answers. If you want to claim to be a theoretical physicist, you must be able to give the rigorous mathematical solution as well.

1.5.2 *The local scene*

The influence of our environment applies to different physics departments, as well as to different countries. If we consider the term ‘theoretical physics’, there is a wide range of opinion as to what constitutes theoretical physics as opposed to physics. It is a fact that in the Cavendish Laboratory in Cambridge, most of the lecture courses are strongly theoretically biased. By this I mean that these courses aim to provide students with a solid foundation in basic theory and its development and relatively less attention is paid to matters of experimental technique. If experiments are alluded to, the emphasis is generally upon the results rather than the experimental ingenuity by which the experimental physicists came to their answers. Although we now give courses on the fundamentals of experimental physics, we expect students to acquire most of their experimental training through practical experiments. This is in strong contrast to the nature of the Cambridge physics courses in the early decades of the twentieth century, which were strongly experimental in emphasis.

Members of departments of theoretical physics or applied mathematics would claim, however, that they teach much ‘purer’ theoretical physics than we do. In their undergraduate teaching, I believe this is the case. There is by definition a strong mathematical bias in the

teaching of these departments, and they are often much more concerned about rigor in their use of mathematics than we are. In other physics departments, the bias is often towards experiment rather than theory. I find it amusing that some members of the Cavendish Laboratory who are considered to be ‘experimentalists’ within the department are regarded as ‘theorists’ by other physics departments in the UK!

The reason for discussing this issue of the local environment is that it can produce a somewhat biased view of what we mean by physics and theoretical physics. My own perspective is that ‘physics’ and ‘theoretical physics’ are part of a continuum of approaches to physical understanding – they are different ways of looking at the same body of material. This is one of the reasons our final-year courses are entitled ‘Experimental and theoretical physics’. In my opinion, there are great advantages in developing mathematical models in the context of the experiments, or at least in an environment where day-to-day contact occurs naturally with those involved in the experiments.

1.6 The plan of the book

This book consists of seven *case studies*, each designed to cover major areas of physics and key advances in theoretical understanding. The case studies are entitled:

- I The origins of Newton’s laws of motion and of gravity
- II Maxwell’s equations
- III Mechanics and dynamics – linear and non-linear
- IV Thermodynamics and statistical physics
- V The origins of the concept of quanta
- VI Special relativity
- VII General relativity and cosmology

These topics have a very familiar ring, but they are treated from a rather different perspective as compared with the standard textbooks – that is why the subtitle of this book is *An alternative view of theoretical reasoning in physics*. My aim is not just to explore the content of the topics but also to recreate the intellectual background to some of the greatest discoveries in theoretical physics.

At the same time, we can gain from such historical case studies important insights into the process of how real physics and theoretical physics are carried out. Such insights can convey some of the excitement and intense intellectual struggle involved in achieving new levels of physical understanding. In a number of these case studies, we will follow the processes of discovery by the same routes followed by the scientists themselves, using only the mathematical techniques available to scientists at the time. For example, we cannot cut corners by assuming we can represent electromagnetic waves by photons until after the discovery of quanta.

In considering each case study, we will also revise many of the basic concepts of physics with which you should be familiar. There are numerous appendices designed to help in areas in which I find students often value additional insight. Finally, each case study is prefaced by a short essay explaining the approach taken and the objectives, which are all

somewhat different and designed to illustrate different aspects of physics and theoretical physics.

1.7 Apologies and words of encouragement

Let me emphasise at the outset that I am not a historian or philosopher of science. I use the history of science very much for my own purpose, which is to illuminate my own experience of how real physicists think and behave. The use of historical case studies is simply a device for conveying something of the reality and excitement of physics. I therefore apologise unreservedly to historians and philosophers of science for using the fruits of their researches, for which I have the most profound respect, to achieve my pedagogical goals. My hope is that students will gain an enhanced appreciation and respect for the works of professional historians of science from what they read in this book.

Establishing the history by which scientific discoveries were made is a hazardous and difficult business; even in the recent past it is often difficult to disentangle what really happened. In my background reading, I have relied heavily upon standard biographies and histories. For me, they have provided vivid pictures of how science actually works and I can relate them to my own research experience. If I have erred in some places, my exculpation can only be the words attributed to Giordano Bruno, ‘Si non e vero, e molto ben trovato’ (if it is not true, it is a very good invention).

My intention is that all advanced undergraduates in physics should be able to profit from this book, whether or not they are planning to become professional theoretical physicists. Although experimental physics can be carried out without a deep understanding of theory, that point of view misses so much of the beauty and stimulation of the subject. Remember, however, the case of Stark, who made it a point of principle to reject almost all theories on which his colleagues had reached a consensus. Contrary to their view, he showed that spectral lines could be split by an electric field, the Stark effect, for which he won the Nobel prize.

Finally, I hope you enjoy this material as much as I do. One of my aims is to put in context all the physics you have met so far and put you into a receptive frame of mind for appreciating the final years of your undergraduate lecture courses. I particularly want to convey a real appreciation of the great discoveries of physics and theoretical physics. These are achievements as great as any in any field of human endeavour.

1.8 References

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